Basics of Neural Network Programming

Binary Classification

deeplearning.ai

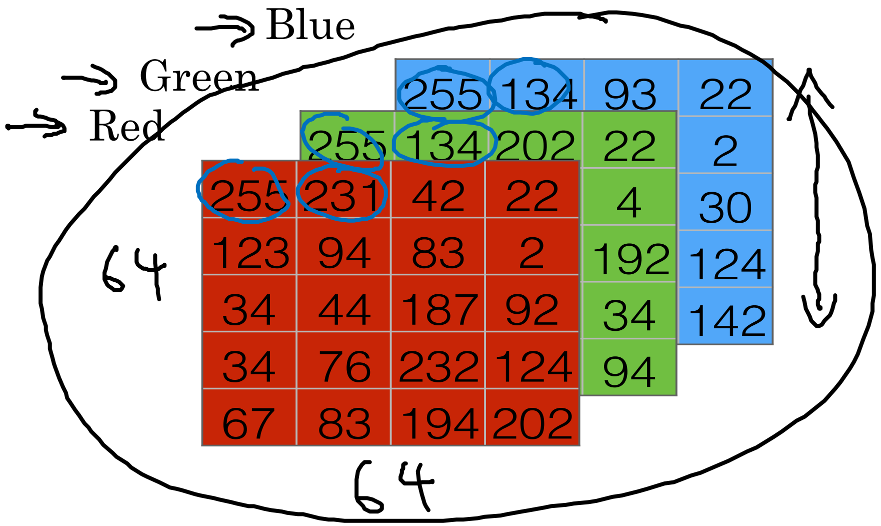
Binary Classification

64

64

→ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$$64 \times 64 \times 3 = 12288$$

$$n = n_x = 12288$$

$$x \longrightarrow y$$

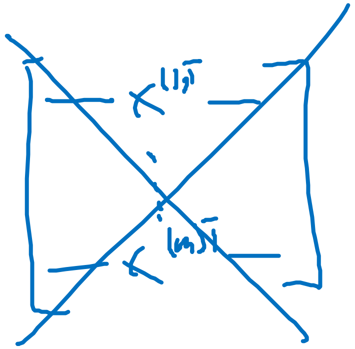
Notation

(x, y) $\quad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

$M = M_{train}$ $\qquad M_{test}$ = #test examples.

$$X = \underbrace{\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}}_{m} \Big\updownarrow n_x$$



$$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$$

$X \in \mathbb{R}^{n_x \times m}$ $\qquad X.\text{shape} = (n_x, m)$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

Basics of Neural Network Programming

Logistic Regression

deeplearning.ai

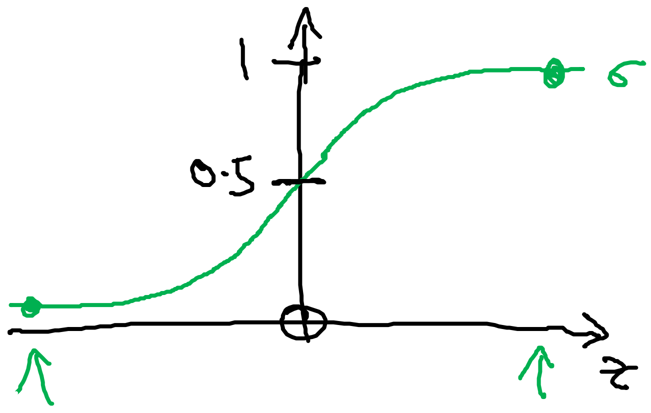
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\quad 0 \le \hat{y} \le 1$

Parameters: $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1, \quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{matrix} \} b \leftarrow \\ \\ \} w \leftarrow \\ \\ \end{matrix}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

Andrew Ng

Basics of Neural Network Programming

Logistic Regression cost function

deeplearning.ai

Logistic Regression cost function

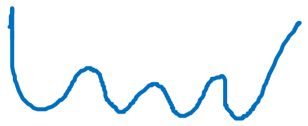
$\rightarrow \hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$z^{(i)} = w^T x^{(i)} + b$

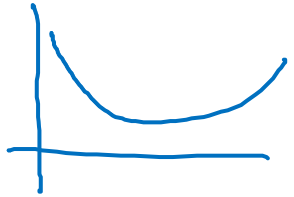
Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$, $y^{(i)}$, $z^{(i)}$ — i-th example.

Loss (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$



$$\mathcal{L}(\hat{y}, y) = -(y \log \hat{y} + (1-y)\log(1-\hat{y}))$$



If $y = 1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y = 0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← Want $\log 1-\hat{y}$ large …. want $\hat{y}$ small

Cost function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

Andrew Ng

Basics of Neural Network Programming

Gradient Descent

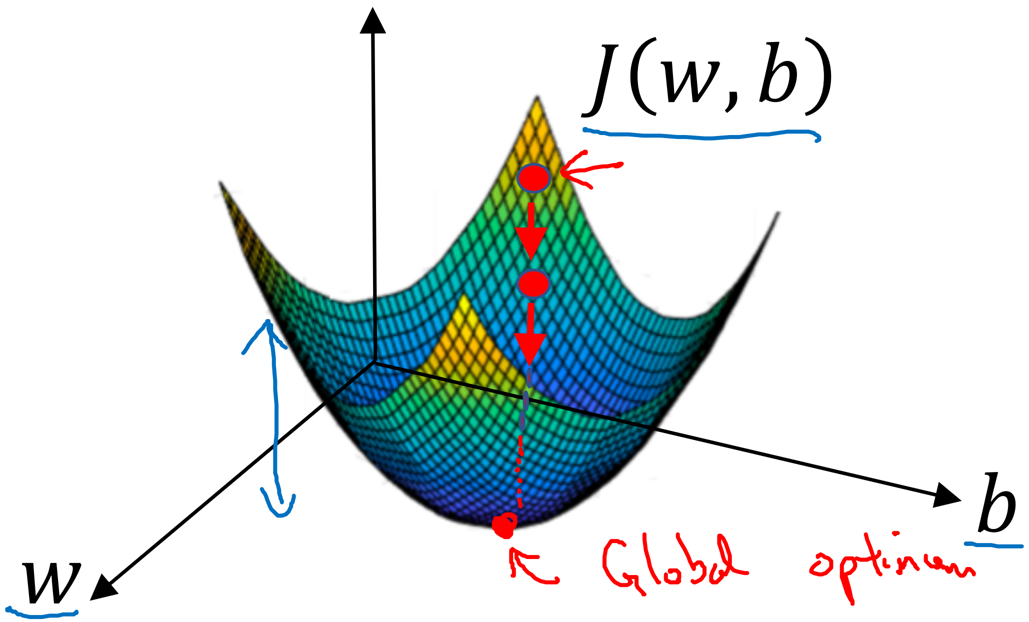
deeplearning.ai

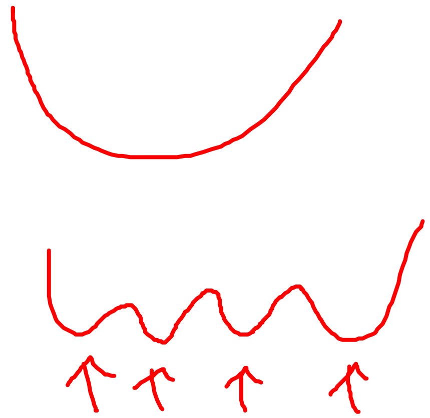
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

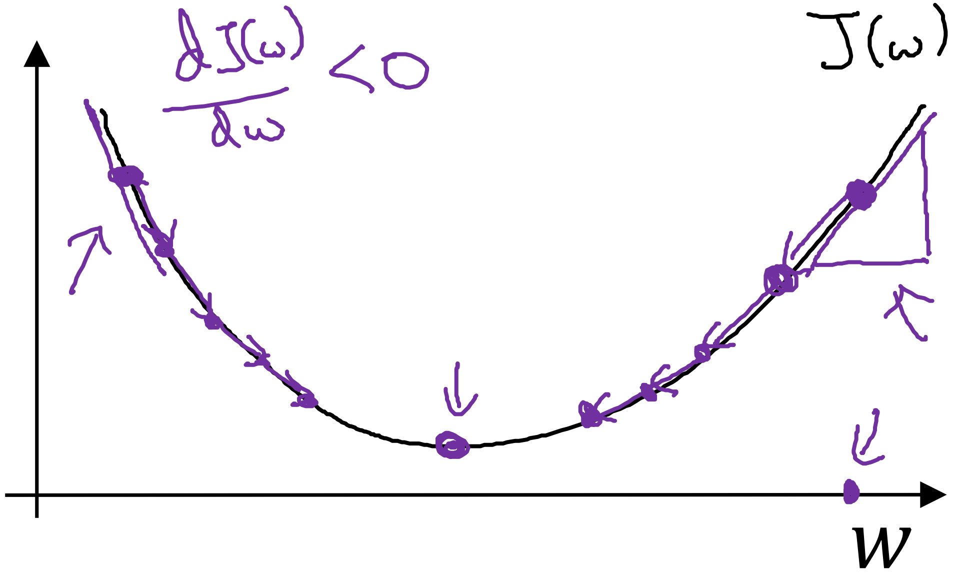
Want to find w, b that minimize $J(w,b)$





Andrew Ng

Gradient Descent



Repeat {

$$w := w - \alpha \frac{dJ(w)}{dw}$$

}

learning rate (α)

"dw"

$$w := w - \alpha \, dw$$

$$\frac{dJ(w)}{dw} = ?$$

$J(w, b)$

$$w := w - \alpha \frac{dJ(w,b)}{dw} \qquad \frac{\partial J(w,b)}{\partial w} \rightarrow dw$$

$$b := b - \alpha \frac{dJ(w,b)}{db} \qquad \frac{\partial J(w,b)}{\partial b} \rightarrow db$$

∂ "partial derivative"

d J

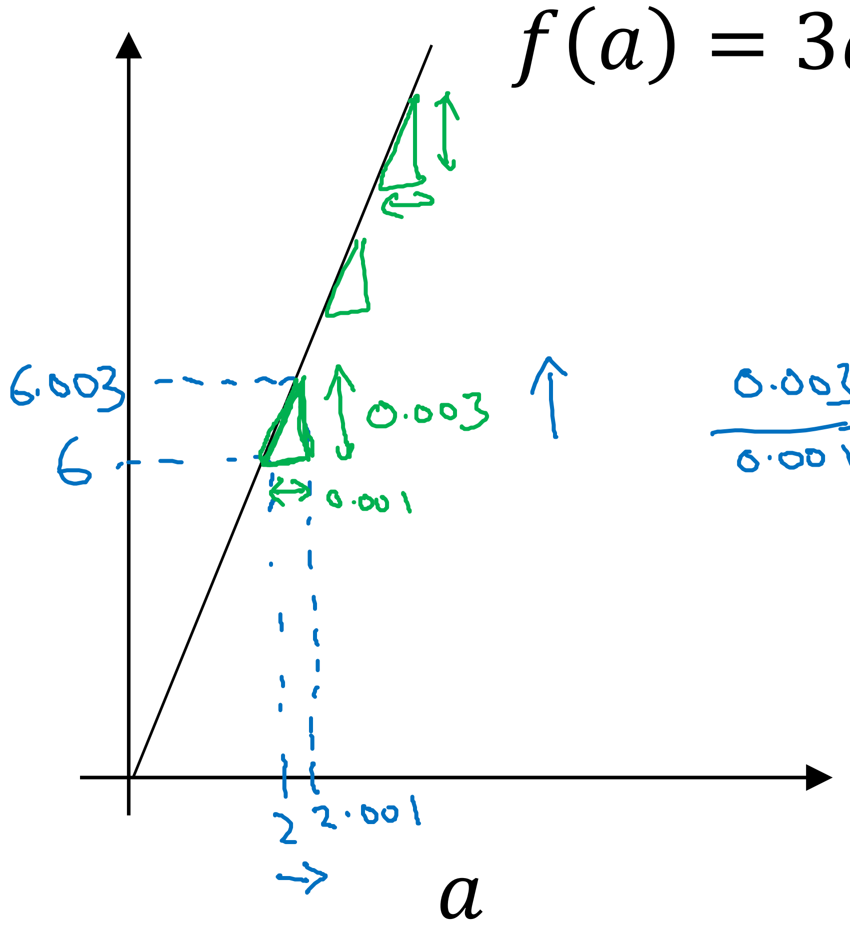
Andrew Ng

Basics of Neural Network Programming

Derivatives

deeplearning.ai

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$ → slope (derivative) of $f(a)$ at $a=2$ is 3

→ $a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

→ $a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a=5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001

0.00000001

0.0000000001

Andrew Ng

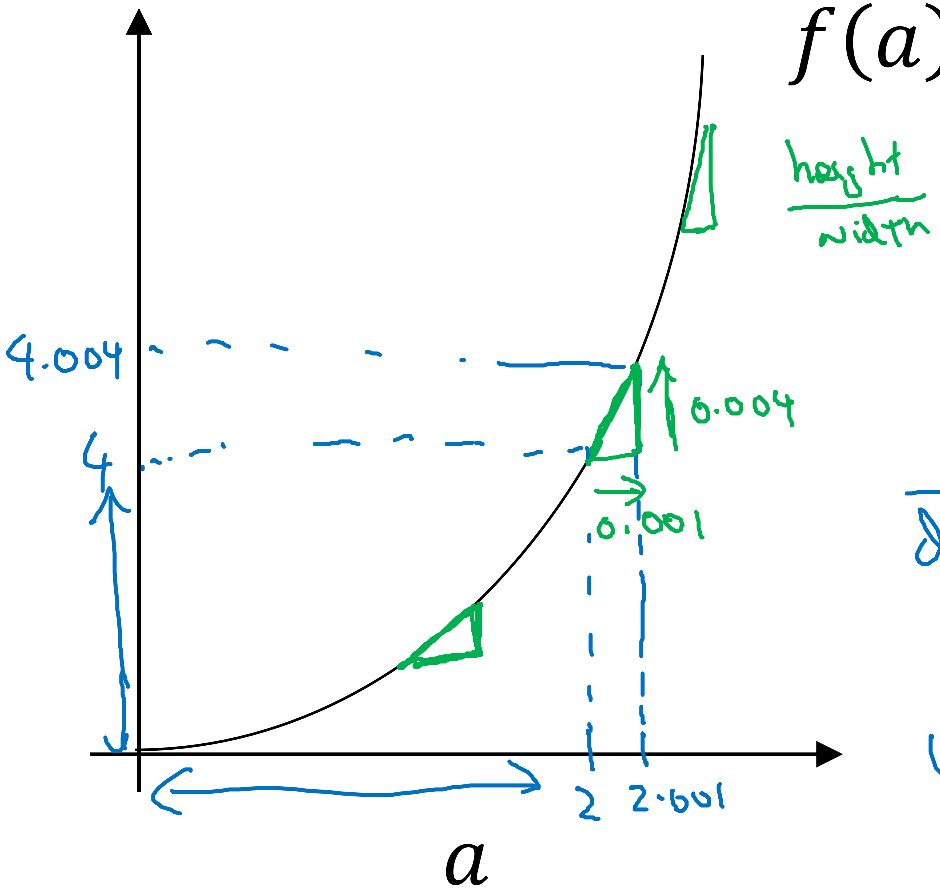
deeplearning.ai

Basics of Neural Network Programming

More derivatives examples

Intuition about derivatives

$f(a) = a^2$



0.001

0.000000 01

$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

(4.004001)

slope (derivative) of f(a) at a=2 is 4.

$$\frac{d}{da} f(a) = 4 \quad \text{when } a = 2.$$

$a = 5$ $\quad f(a) = 25$

$a = 5.001$ $\quad f(a) \approx 25.010$

$$\frac{d}{da} f(a) = 10 \quad \text{when } a = 5$$

$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = 2a$$

Andrew Ng

More derivative examples

$f(a) = a^2$ $\quad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\quad a = 2 \quad f(a) = 4$

$a = 2.001 \quad f(a) \approx 4.004$

$f(a) = a^3$ $\quad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$ $\quad a = 2 \quad f(a) = 8$

$a = 2.001 \quad f(a) \approx 8.012$

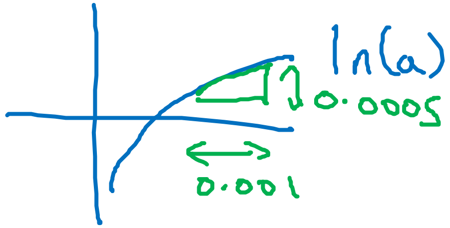
$f(a) = \log_e(a)$

$\ln(a)$

$\frac{d}{da} f(a) = \frac{1}{a}$ $\quad a = 2 \quad f(a) \approx 0.69315$

$a = 2.001 \quad f(a) \approx 0.69365$

$0.0005 \quad 0.0005$



$\frac{d}{da} f(a) = \frac{1}{2}$

Andrew Ng

deeplearning.ai

Basics of Neural Network Programming

Computation Graph

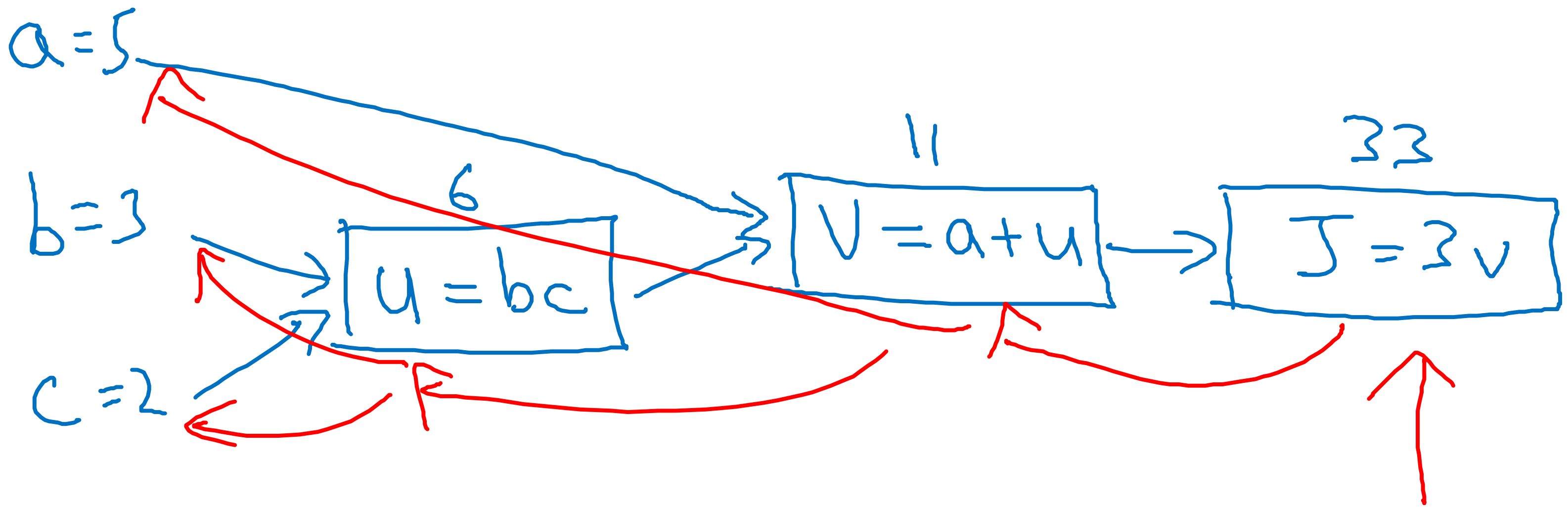
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a + \underbrace{bc}_{u}}_{v})}_{J} = 3(5 + 3 \times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$

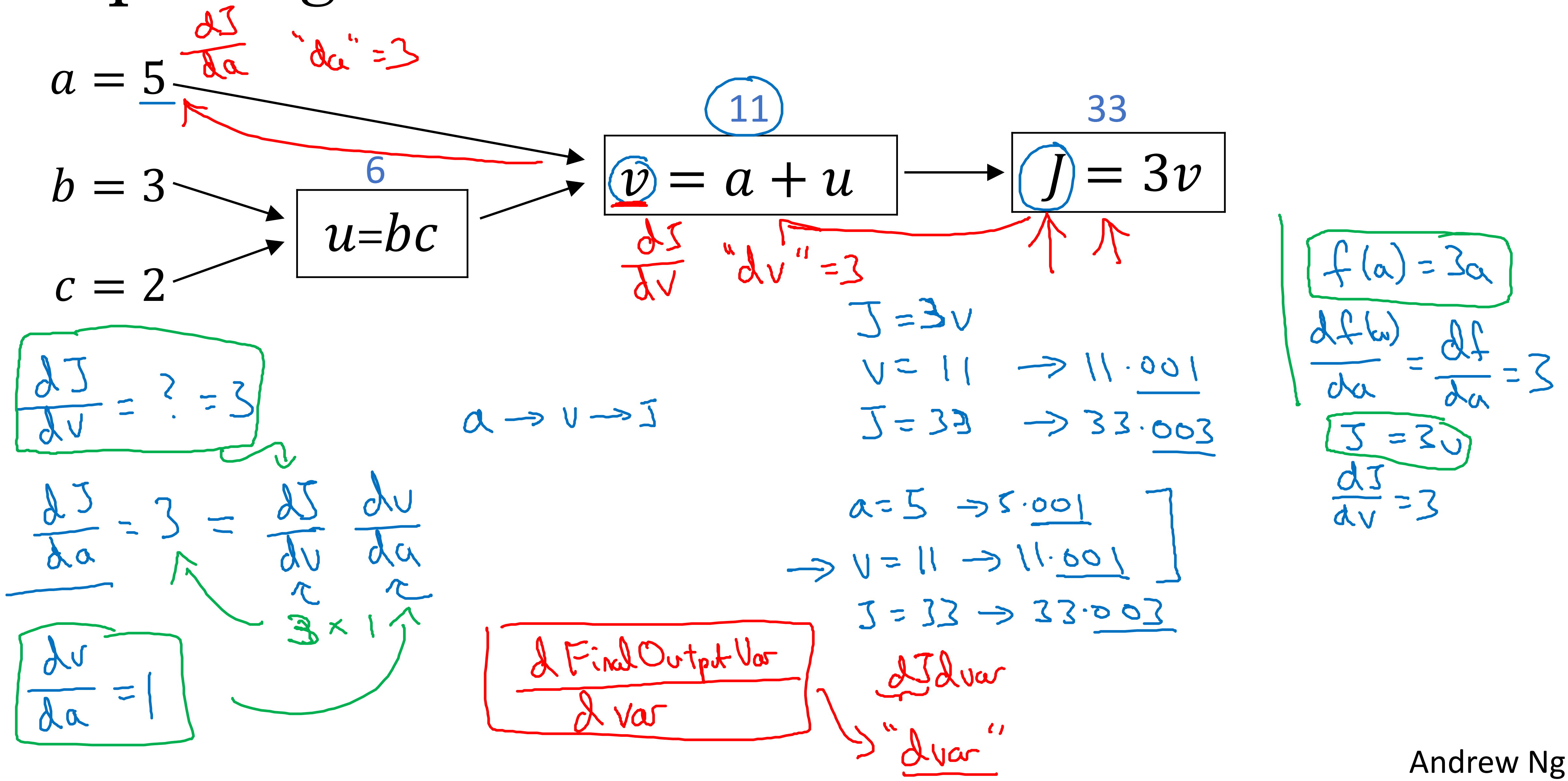


deeplearning.ai

Basics of Neural Network Programming

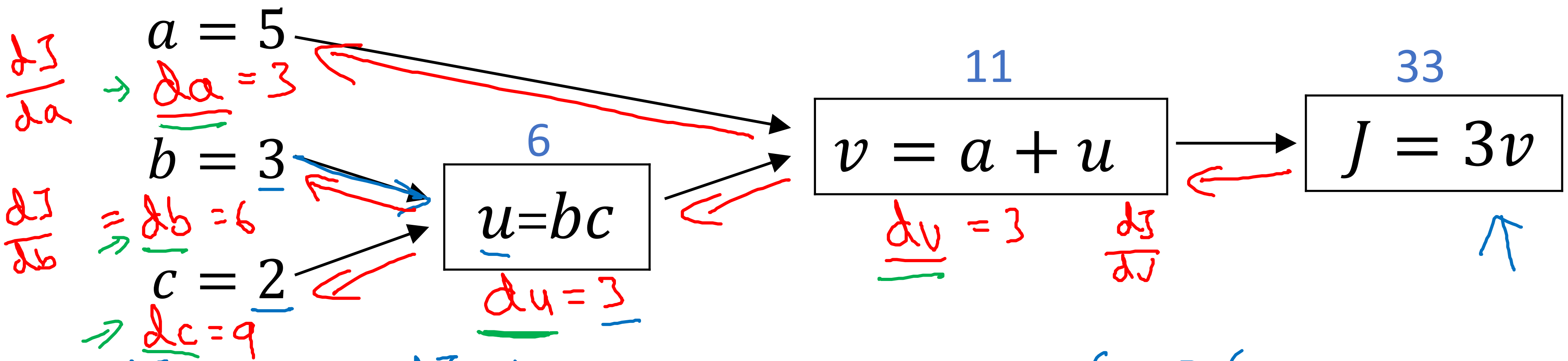
Derivatives with a Computation Graph

Computing derivatives



Andrew Ng

Computing derivatives



$$\frac{dJ}{du} = 3 = \frac{dJ}{dv} \cdot \frac{dv}{du}$$ (3 · 1)

$$\frac{dJ}{db} = \frac{dJ}{du} \cdot \frac{du}{db} = 6$$ (3 · 2)

$$\frac{dJ}{da} = \frac{dJ}{du} \cdot \frac{du}{da} = 9$$ (3 × 3)

$u = 6 \rightarrow 6.001$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$

$u = b \cdot c = 6 \rightarrow 6.002$

$J = 33.006$

$c = 2$

$.006$

$v = 11.002$

$J = 3v$

deeplearning.ai

Basics of Neural Network Programming

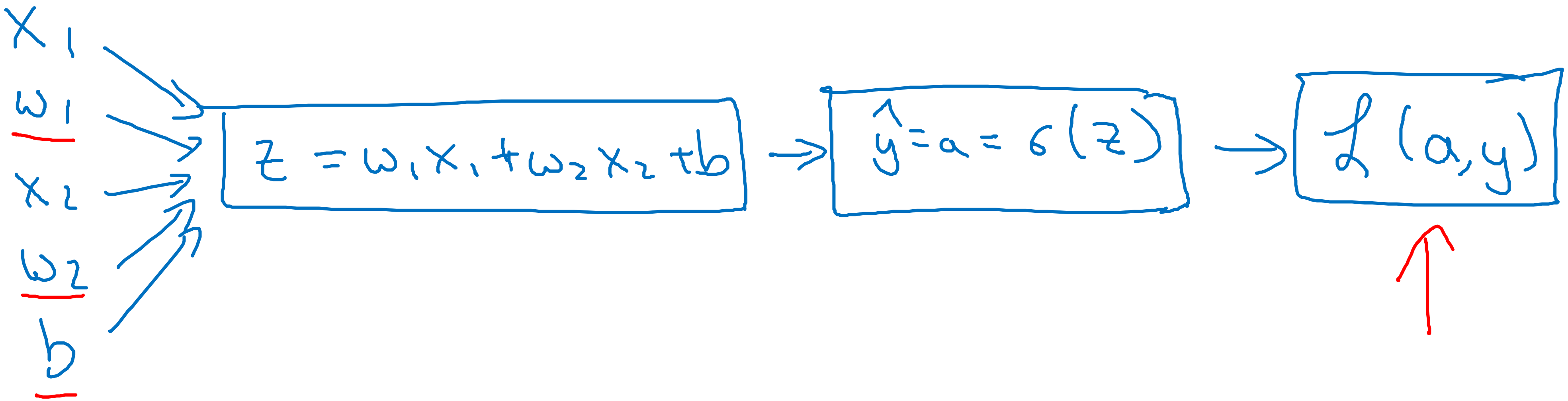
Logistic Regression Gradient descent

Logistic regression recap

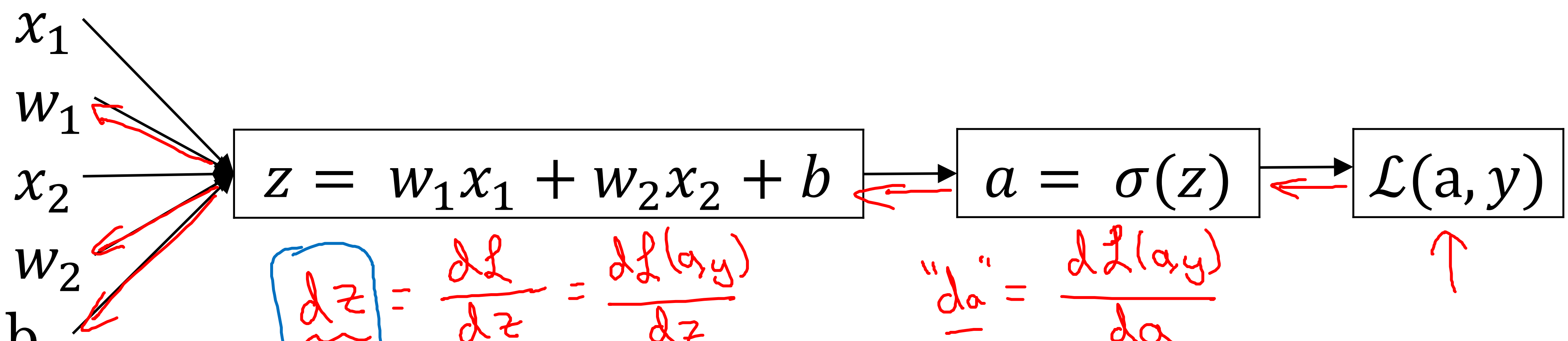
$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives



$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz} = a - y = \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$$

$$\text{"}da\text{"} = \frac{d\mathcal{L}(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{da}{dz} = a(1-a)$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$
$$w_2 := w_2 - \alpha\, dw_2$$
$$b := b - \alpha\, db.$$

Andrew Ng

deeplearning.ai

Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$(x^{(i)}, y^{(i)})$$

$$dw_1^{(i)},\ dw_2^{(i)},\ db^{(i)}$$

$$\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})}_{dw_1^{(i)}}$$

$$dw_1^{(i)} - (x^{(i)}, y^{(i)})$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

→ For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$ ($n=2$)

$db \mathrel{+}= dz^{(i)}$

dw_3 ⋮ dw_n

$J /= m$ ←

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$ ←

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

Vectorization

Andrew Ng

Basics of Neural Network Programming

Vectorization

deeplearning.ai

What is vectorization?

$z = \underbrace{w^T x} + b$

$w = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} \quad x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} \quad w \in \mathbb{R}^{n_x} \quad x \in \mathbb{R}^{n_x}$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

$z = \underbrace{np.dot(w, x)}_{w^T x} + b$

→ GPU
→ CPU

} SIMD – single instruction multiple data.

deeplearning.ai

Basics of Neural Network Programming

More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$$u = Av$$

$$u_i = \sum_i \sum_j A_{ij} v_j$$

```
u = np.zeros((n,1))
for i ...
    for j ...
        u[i] += A[i][j] * v[j]
```

```
u = np.dot(A,v)
```

Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v, 0)
v**2        1/v
```

Logistic regression derivatives

$J = 0$, ~~$dw1 = 0$, $dw2 = 0$~~, $db = 0$ dw = np.zeros((n_x, 1))

→ for i = 1 to n:

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -\left[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right]$$

$$dz^{(i)} = a^{(i)}(1 - a^{(i)})$$

for $j = 1 \ldots n_x$: $dw_j \mathrel{+}= \ldots$

~~$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$~~

~~$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$~~ $n_x = 2$ $dw \mathrel{+}= x^{(i)} dz^{(i)}$

$$db \mathrel{+}= dz^{(i)}$$

$J = J/m$, ~~$dw_1 = dw_1/m$, $dw_2 = dw_2/m$~~, $db = db/m$

$dw \mathrel{/}= m.$

Andrew Ng

deeplearning.ai

Basics of Neural Network Programming

Vectorizing Logistic Regression

Vectorizing Logistic Regression

$$z^{(1)} = w^T x^{(1)} + b \qquad z^{(2)} = w^T x^{(2)} + b \qquad z^{(3)} = w^T x^{(3)} + b$$

$$a^{(1)} = \sigma(z^{(1)}) \qquad a^{(2)} = \sigma(z^{(2)}) \qquad a^{(3)} = \sigma(z^{(3)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \qquad (n_x, m) \qquad \mathbb{R}^{n_x \times m}$$

$$w^T \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$Z = [z^{(1)} \; z^{(2)} \cdots z^{(m)}] = w^T X + [b \; b \cdots b] = [w^T x^{(1)} + b \quad w^T x^{(2)} + b \cdots w^T x^{(m)} + b]$$

$1 \times m$

$$Z = np.dot(w.T, X) + b$$

$(1, 1)$ $\mathbb{R}$ "Broadcasting"

$$A = [a^{(1)} \; a^{(2)} \cdots a^{(m)}] = \sigma(Z)$$

Andrew Ng

deeplearning.ai

Basics of Neural Network Programming

Vectorizing Logistic Regression's Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$ $\quad \ldots$

$dZ = [dz^{(1)} \; dz^{(2)} \; \ldots \; dz^{(m)}]$ $\quad 1 \times m$

$A = [a^{(1)} \ldots a^{(m)}]$ $\quad Y = [y^{(1)} \ldots y^{(m)}]$

$\rightarrow dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \ldots]$

$\rightarrow dw = 0$

$dw \mathrel{+}= x^{(1)} dz^{(1)}$

$dw \mathrel{+}= x^{(2)} dz^{(2)}$

$\vdots$

$dw \mathrel{/}= m$

dw_1, dw_2, $\ldots$

$db = 0$

$db \mathrel{+}= dz^{(1)}$

$db \mathrel{+}= dz^{(2)}$

$\vdots$

$db \mathrel{+}= dz^{(m)}$

$db \mathrel{/}= m.$

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$

$$= \frac{1}{m} \text{np.sum}(dZ)$$

$$dw = \frac{1}{m} X dZ^T$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \ldots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[x^{(1)} dz^{(1)} + \ldots + x^{(m)} dz^{(m)} \right]$$

$n \times 1$

Andrew Ng

Implementing Logistic Regression

$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$

for i = 1 to m:

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$$

$$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$$

$dw \mathrel{+}= x^{(i)} * dz^{(i)}$

$$db \mathrel{+}= dz^{(i)}$$

$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$

$db = db/m$

for iter in range(1000):

$Z = w^T X + b$

$= np.dot(w.T, X) + b$

$A = \sigma(Z)$

$dZ = A - Y$

$dw = \frac{1}{m} X dZ^T$

$db = \frac{1}{m} np.sum(dZ)$

$w := w - \alpha dw$

$b := b - \alpha db$

Andrew Ng

deeplearning.ai

Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

	Apples	Beef	Eggs	Potatoes
Carb	56.0	0.0	4.4	68.0
Protein	1.2	104.0	52.0	8.0
Fat	1.8	135.0	99.0	0.9

= A (3,4)

59 cal

$\frac{56}{59} \approx 94.9\%$

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

```
cal = A.sum(axis = 0)
percentage = 100*A/(cal.reshape(1,4))
```

(3,4) / (1,4)

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + 100 \quad \rightarrow \quad \begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}100\\100\\100\\100\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 200 & 300\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}100 & 200 & 300\\100 & 200 & 300\end{bmatrix}$$

(m,n) $(2,3)$ $\quad$ $(1,n) \rightsquigarrow (m,n)$ $\quad$ $(2,3)$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100\\200\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}100 & 100 & 100\\200 & 200 & 200\end{bmatrix} =$$

(m,n) $\quad$ $(m,1) \rightarrow (m,n)$

General Principle

(m, n) matrix $\quad + \; - \; * \; / \quad (1, n) \rightsquigarrow (m, n)$

$\qquad\qquad\qquad\qquad\qquad\quad (m, 1) \rightsquigarrow (m, n)$

$(m, 1) \quad + \quad \mathbb{R}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 & 102 & 103 \end{bmatrix}$$

Matlab/Octave: bsxfun

deeplearning.ai

Basics of Neural Network Programming

Explanation of logistic regression cost function (Optional)

Logistic regression cost function

$\hat{y} = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

Interpret $\hat{y} = P(y=1 \mid x)$

If $y=1$: $P(y|x) = \hat{y}$

If $y=0$: $P(y|x) = 1 - \hat{y}$

Logistic regression cost function

→ If $y = 1$: $p(y|x) = \hat{y}$

→ If $y = 0$: $p(y|x) = 1 - \hat{y}$

$p(y|x)$

$$p(y|x) = \hat{y}^{y} (1-\hat{y})^{(1-y)} \leftarrow$$

If $y=1$: $p(y|x) = \hat{y} \; (1-\hat{y})^{0}$ $= 1$

If $y=0$: $p(y|x) = \hat{y}^{0} (1-\hat{y})^{(1-y)} = 1 \times (1-\hat{y}) = 1-\hat{y}$

$\hat{y}^{0} = 1$

$$\uparrow \log p(y|x) = \log \hat{y}^{y} (1-\hat{y})^{(1-y)} = y \log \hat{y} + (1-y) \log (1-\hat{y})$$

$$= - \mathcal{L}(\hat{y}, y) \downarrow$$

Andrew Ng

Cost on m examples

$$\log p(\text{labels in training set}) = \log \prod_{i=1}^{m} p(y^{(i)} | x^{(i)})$$

$$\log p(\ldots\ldots) = \sum_{i=1}^{m} \underbrace{\log p(y^{(i)} | x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)}, y^{(i)})}$$

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Maximum likelihood estimation

Cost: (minimize) $J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$