

Tour Guide: Deep Learning in Trajectory Optimization

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Summary

- Real-time trajectory optimization for autonomous vehicles requires solving optimization problems **fast**
- Solver runtime can be reduced by providing a good initial guess to the solution ("warm-start")
- We train a fully-connected neural network on a family of optimal trajectories for a reusable launch vehicle (RLV)
- We achieve a 2.2x faster solver by warm-starting with our neural network

Related Work

- Using deep learning to "optimize the optimizers" is a relatively new research thread
- (Mansard, 2018) trains a neural network offline to provide warm-start solutions for a UAV trajectory planner
- RLV problem is higher dimension, longer time horizon, and more challenging dynamics

Data/Features/Models

- 14,000 optimal trajectories for how the RLV should travel from initial altitude to a final latitude & longitude

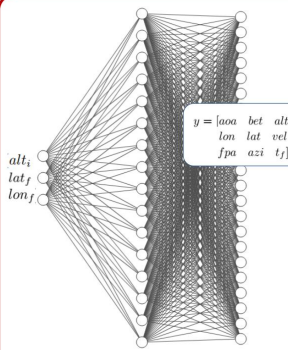
$$x = [alt_i \quad lat_f \quad lon_f] \in \mathbb{R}^3$$

$$y = [aoa \quad bet \quad alt \quad lon \quad lat \quad vel \quad fpa \quad azi \quad t_f] \in \mathbb{R}^{801}$$

- Generated with GPOPS-II optimal control software (12 hours)
- Data shows clear trends \rightarrow Find a low-dimensional representation for easy implementation on flight computer
- Multivariate regression problem. Penalize mean squared error

$$J(y, \hat{y}) = \frac{1}{m} \frac{1}{n_y} \sum_{i=1}^m \sum_{j=1}^{n_y} (y_j - \hat{y}_j)^2$$

- Used 90/10 train/test split. Data shifted and scaled on both input and output (to ensure equal weighting in cost function)
- TensorFlow + Keras for neural network training using ADAM



Results & Discussion

- To ensure small computation footprint, searched for minimum number of hidden units necessary to achieve $< 1\%$ error on train/test set
- Final architecture: Single hidden layer with 16 ReLUs (very small!)

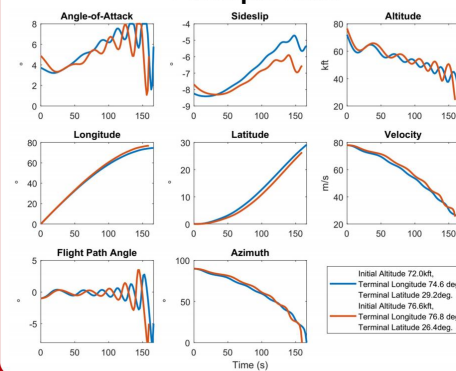
	Min. Loss	Max. Loss	Mean Loss
Train	0.0007	0.018	0.0077
Test	0.0007	0.016	0.0071

- Utilized neural network to warm-start solver on 1000 previously unseen problems \rightarrow 2.2x faster than cold-start

	Minimum (s)	Maximum (s)	Mean (s)
Cold-Start	0.64	8.08	1.76
Warm-Start (w/Neural Network)	0.25	1.31	0.77
$\Delta =$ Cold Start - Warm Start	0.09	6.96	1.00

- Promising result. Future work will build upon this framework to tackle higher dimensional problems.

Example Data



References

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Presentation video & code available at

https://github.com/guthriejd1/cs230_project