Predicting

- Options are financial derivatives that gives the right to buy (call) or sell (put) a security at or before a certain date.
- The value of a European option (right to buy/sell at a certain date, and not before) can be modeled with the commonly-used Black-Scholes formula [1]:
  \[ C = S \Phi(d_1) - X e^{-rT} \Phi(d_2) \]
- This model makes many assumptions (especially of volatility) and often mismatches empirical findings.
- We use deep learning to construct models that try to price options using historical data.
- With MLP and LSTM architectures we are able to significantly outperform the Black-Scholes model.

Features

- Out of 5 features, we had 4 raw features (days until expiry, strike price, risk-free rate, and underlying security price) and 1 derived feature, volatility.
- From the bid and ask prices, we also computed the equilibrium price which we use as an alternative label.
- We used a 98-1-1 train/dev/test split.

Models

- For our problem, we explore three network architectures: two multilayer perceptrons and one LSTM.
  - MLP1
    - 4-layer NN with three 400-neuron hidden layers and one output layer with one neuron
    - Uses 20-day historical volatility as an additional input
    - Uses equilibrium price as the label
  - MLP2
    - Same architecture as MLP1, but uses multitask learning to predict both bid and ask prices.
  - LSTM
    - Feeds 20 timesteps of closing history through LSTM layers, and merges LSTM output with rest of features into fully-connected layers to predict equilibrium price

Results

- All three models outperform Black-Scholes, with MLP2 performing the best.

<table>
<thead>
<tr>
<th>Model</th>
<th>train MSE</th>
<th>MSE</th>
<th>Bias</th>
<th>AAPE</th>
<th>MAPE</th>
<th>PE5</th>
<th>PE10</th>
<th>PE20</th>
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<tbody>
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</table>

Discussion

- Our models performs significantly better than the Black-Scholes model, with MLP2 performing the best.
  - Multitask learning is shown to be very effective in predicting bid/ask prices because they are similar tasks
  - Our LSTM did not perform as well as either MLP.
  - We believe that we can do better on this task by performing a more thorough hyperparameter search (e.g. number of timesteps)
- Instead of making assumptions about financial mechanics as in the Black-Scholes model, our deep learning approach learns only from historical data, and seems to be a very promising way to forecast options prices.

Future Work

- If given more time, we would like to isolate characteristics such as time until expiry, etc. and conduct deeper error analyses to determine if our models perform differently on contracts with different characteristics.
- Given more time to train models, we would like to refine our LSTM model by increasing the number of timesteps so that we may better predict volatility.
- We would also like to train models on the reverse problem of finding the volatility implied by a given option price.
- Additionally, our findings can be applied toward pricing exotic options, e.g. binary or Asian options.

References