Estimating the Discrete Fourier Transform using Deep Learning

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Motivation

- Entire fields hinge upon **the Fourier Transform** and its **efficient computation**
- Faster implementations of the **Discrete Fourier Transform (DFT)** allow for more efficient computation in a wide variety of systems, such as **medical imaging**, **optics**, and **radar systems.**
- Neural network architectures may be the solution to **faster DFT computation times**.

The Discrete Fourier Transform (DFT)

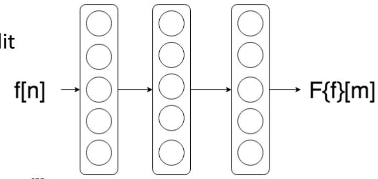
- N-point Discrete Fourier Transform

$$F\{f\}[m] = \sum_{n=0}^{N-1} f[n] e^{2\pi i m n/N}, \quad m = 0, \ldots, N-1.$$

- Maps N-vector to N-vector
- Generally used to **map time series signals** to their **frequency domain** representation
- Can be represented as a **dense (complex-valued) matrix multiply**
- Naive computation time: **$O(N^2)$**
- Fast implementation (Fast Fourier Transform (FFT)): **O(Nlog(N))**
- There **does not currently exist** a general **algorithm that implements the DFT faster** than O(Nlog(N))

Approach

- Three fully connected layers, linear activation functions
- Training/Test Data
  - 30000 random signals, bandlimited to 10 Hz (to avoid aliasing)
  - With/without noise
  - 90/10 training/test split

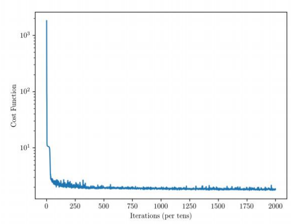


- Cost Function

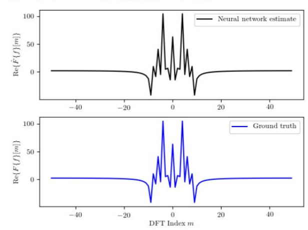
$$\mathcal{J} = (1/m) \sum_{i=1}^{m} \|F\{f_i\} - \hat{F}\{f_i\}\|_2^2$$

Experimental Results

- Training Error = 8.1 * 10^{-4}, Test Error = 2.1 * 10^{-2}
- Naive DFT computation time = 4.1μs, FFT computation time = 3.5μs, **neural network DFT computation time = 1.9μs**
- Neural network **successfully estimates DFT** well (see below for example)
- Empirically, architecture is **2.2x faster than naive computation and 1.8x faster than FFT** for N = 100.



$$\|F\{f\} - \hat{F}\{f\}\|_2 / \|F\{f\}\|_2 = 6.0 \times 10^{-4}$$



Hyperparameter Selection

- 17 nodes per (hidden) layer
- Training epochs = 20000
- Learning rate = 0.001
- Minibatch size = 250
- Drop-out probability = 0.9
- Other regularization was found to not improve performance

Future Work

- Exploiting structure in signals
  - Sparsity (compressed sensing)
- Other transforms
  - Discrete Cosine transform
  - Radon transform
  - Continuous Wavelet transform

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